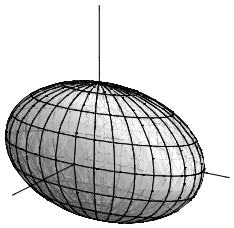
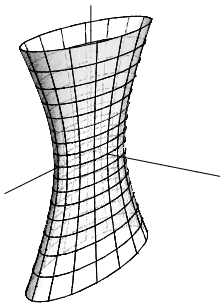
Quadric Surfaces

Six basic types of quadric surfaces:

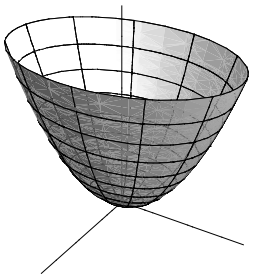
- ellipsoid
- cone
- elliptic paraboloid
- hyperboloid of one sheet
- hyperboloid of two sheets
- hyperbolic paraboloid



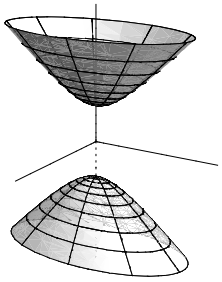
(A)



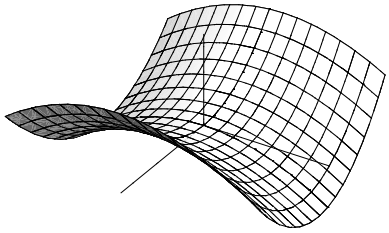
(B)



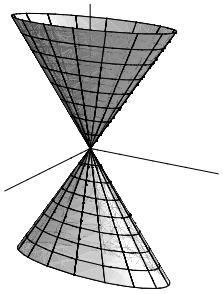
(C)



(D)



(E)



(F)

Quick reminder:

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = r$ describes ...
  - an ellipse if $r > 0$.
  - a point if $r = 0$ (we consider this a "degenerate" ellipse).
  - nothing if $r < 0$.
- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = r$ describes ...
  - a hyperbola if $r \neq 0$.
  - a pair of lines if $r = 0$ (we consider this a "degenerate" hyperbola).
- $y = ax^2 + b$ describes a parabola.

1. For each surface, describe the traces of the surface in $x = k$, $y = k$, and $z = k$. Then pick the term from the list above which seems to most accurately describe the surface (we haven't learned any of these terms yet, but you should be able to make a good educated guess), and pick the correct picture of the surface.

   (a) $\dfrac{x^2}{9} - \dfrac{y^2}{16} = z$.

   - Traces in $x = k$:
   - Traces in $y = k$:
   - Traces in $z = k$:

   (b) $\dfrac{x^2}{4} + \dfrac{y^2}{25} + \dfrac{z^2}{9} = 1$.

   - Traces in $x = k$:
   - Traces in $y = k$:
   - Traces in $z = k$:

   (c) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = \dfrac{z}{2}$.

   - Traces in $x = k$:
   - Traces in $y = k$:
   - Traces in $z = k$:

   (d) $\dfrac{z^2}{4} - x^2 - \dfrac{y^2}{4} = 1$.

   - Traces in $x = k$:
   - Traces in $y = k$:
   - Traces in $z = k$:

   (e) $x^2 + \dfrac{y^2}{9} = \dfrac{z^2}{16}$.

   - Traces in $x = k$:
   - Traces in $y = k$:
   - Traces in $z = k$:

(f) $\dfrac{x^2}{9} + y^2 - \dfrac{z^2}{16} = 1.$

- Traces in $x = k$:
- Traces in $y = k$:
- Traces in $z = k$:

2. Sketch the surface $9y^2 + 4z^2 = 36$. What type of quadric surface is it?

3. Sketch the surface $y^2 + 2y + z^2 = x^2$. What type of quadric surface is it?

4. What type of quadric surface is $4x^2 - y^2 + z^2 + 9 = 0$?